Radiant Heating of Semitransparent Materials[1]

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The problem of a semitransparent solid heated by a combination of convective and radiative inputs is treated. Analytic solutions are obtained for two special cases. First, a solution is given for the steady-state case of ablation in a semi-infinite bar. Second, another solution is given for the transient response of a nonablating semi-infinite bar to a constant radiative and convective heat input. Numerical results are obtained for both problems for pure radiant heating, and the effect of varying optical parameters is discussed.

Nomenclature

H_w = air enthalpy at wall
H_∞ = enthalpy of freestream
K_n = integration constants
$\dot{q}_c$ = convective heat flux to a cold wall
s = ablated length (Fig. 1)
ϵ = effective emissivity
σ = Stefan-Boltzmann constant
F_x = incident radiant flux
k = thermal conductivity
c_p = specific heat
q^* = ablation parameter
Δh_v = heat of vaporization
η = transpiration parameter
ϕ,τ = Laplace transformed time and temperature
T = temperature
x = position coordinate
a = attenuation coefficient
α = thermal diffusivity
ρ = density
R = diffuse reflectance

An ever increasing number of situations are being encountered in the missile industry where radiant heating is a significant fraction of the total heat input. Many of the materials used for thermal protection are semitransparent; consequently a sizeable fraction of the incident radiant flux may flow through the material heating it from within. This will result in an earlier temperature response in depth than would otherwise be the case. As a consequence thermal protection systems designed for short time performance may be adversely affected.

In other applications, such as the annealing of glass in a furnace, the problem of radiant heating is also important. Indeed, the glass industry has been concerned with the problem since 1930, when Dreosti (1)[4] published a study of transmission and reflection phenomena in opal glass. In 1947, Hamaker (2) published a series of papers dealing with the combined effects of heat conduction and radiation in a scattering medium. This was followed in 1952 by Kellett's paper (3) on heat flow in clear glass heated by a radiant source. Subsequently, a series of papers on the subject by German authors appeared (4–7). In 1956, Gardon (8) investigated the effect of refractive index and multiple internal reflections upon the diffuse transmission, emission, and reflection of thin sheets of clear glass and obtained expressions for the spectral emissivities of isothermal layers of such media. In a subsequent paper (9), he presented a method for calculating temperature distributions in glass plates undergoing heat treatment in a combined radiative and convective environment.

In all of these treatments except the last, consideration was limited to the steady state. In most of them, the primary objective was evaluation of an effective thermal conductivity for use in the Fourier heat conduction equation for calculating temperatures in high temperature glass. Considerable emphasis in most of these works was placed on self-induced thermal radiation and its effect upon the internal temperature distribution. In Ref. 9, a procedure for calculating the transient response of thin clear glass plates undergoing heat treatment in a radiant environment was given. Solutions were obtained by means of a digital computer. None of the works referenced so far included ablation effects.

In 1958 Kadanoff (10) evaluated the effects of self-induced thermal radiation upon the temperature response of an aerodynamically heated ablating medium. His treatment was for the transient response of an absorbing-scattering material but did not treat external radiant heating. Also in 1958, Grigorev (11) treated the problem of radiant heating of an unbounded plate but did not consider absorption of the radiant energy in depth. His results are thereby applicable only to an opaque material, such as a metal.

The problem considered in the present report is that of semitransparent absorbing-scattering material heated by a transient radiant-convective input. The heat equation is formulated so as to include the effect of absorption-in-depth of the radiant energy, and the effects of ablation are included. In order to provide a better understanding of the phenomena and to provide a check for a computer program, the formulation is specialized to the case of a semi-infinite bar heated by a constant combined radiant and convective input. Two analytic solutions to this problem are obtained for the case of a material with constant properties. One solution treats the case of steady-state ablation, and the other is for the transient response prior to ablation. Numerical results are presented and discussed for both solutions. The detailed treatment of emission of radiation is not accounted for in the analysis, since its effects have been extensively discussed in the literature (8, 13–23). Its effect is accounted for in a gross manner in the steady-state ablation solution by use of an effective emissivity. This approximation does not in any way influence the conclusions drawn from the study. It

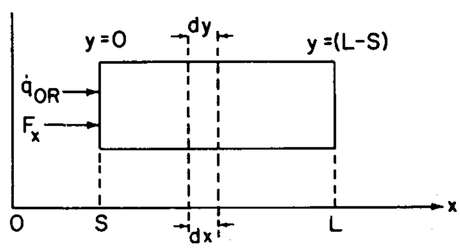

Fig. 1 Geometry schematic

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should be noted that the treatment concerns itself with monochromatic irradiation or a grey medium.

Analysis

Mathematical Formulation

Consider a bar exposed to combined radiant and convective heat inputs as illustrated in Fig. 1. Assuming that the material does not decompose and that one-dimensional energy transport takes place by means of conduction and radiation fluxes, the heat equation in a laboratory coordinate system is

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) - \frac{\partial F}{\partial x} \quad [1]$$

In an absorbing-scattering medium with a perfectly transparent rear boundary, the radiant flux F at any point x may be represented by the following expression:

$$F(x) = (1 - R)F_x e^{-a(x-s)} \quad [2]$$

The quantity R is the diffuse reflectivity of the material to the incident radiant flux F_x. This relation is valid for the region $s < x < L$ and constitutes the definition of the radiant flux attenuation coefficient a. Both the flux attenuation coefficient and the diffuse reflectivity are considered as average values taken over the band of wavelengths of the radiant source. Accordingly, a and R are not only properties of the material but also functions of the spectral distribution of the radiant source. These quantities must be determined from an experimental procedure whereby the diffuse transmission spectrum of several different thicknesses of the material is measured, for example, by spectrophotometric techniques. These spectra are then integrated over the band of wavelengths characteristic of the radiant source to be used, and the integrated transmittance is plotted on a semilog scale as a function of sample thickness, as shown in Fig. 2. The asymptotic slope of this curve is equal to a, and the intercept of this asymptote is $(1 - R)$. Boundary effects due to multiple internal reflection cause deviations from the asymptotic behavior for very thin samples, yielding a higher apparent reflectance and transmittance. These effects are not considered in the present formulation. Accordingly, the results presented herein are applicable only to a very thick sample or to one with a perfectly transparent rear boundary. It should be noted that if one wishes to account for emission one may merely add that term to the right-hand side of Eq. [2].

Eqs. [1] and [2] may be combined, yielding the following differential equation:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + a(1 - R)F_x e^{-a(x-s)} \quad [3]$$

The initial and boundary conditions for a semi-infinite bar at uniform initial temperatures are given by

$$T(x, 0) = T_0 \quad [4]$$

$$T(\infty, t) = T_0 \quad [5]$$

$$\dot{q}_{0r} = -\left(k \frac{\partial T}{\partial x}\right)_{x=s} \quad [6]$$

In Eq. [6], $\dot{q}_{0r}$ is the unsteady aerodynamic heat input corrected for hot wall and radiative emission reductions, the latter being accounted for by the use of an effective emissivity:

$$\dot{q}_{0r} = \dot{q}_c\left(1 - \frac{H_w}{H_\infty}\right) - \epsilon\sigma T_w{}^4 \quad [7]$$

Under the influence of the radiant and aerodynamic heat

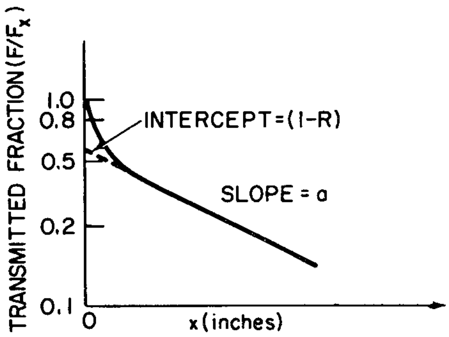

Fig. 2 Transmittance vs thickness

inputs, the bar exhibits a temperature response. It is postulated that, when the surface temperature reaches a specified value T_A, the surface of the bar begins to recede with a velocity $\dot{s}(t)$, and boundary condition [6] changes to the following:

$$T(s,t) = T_A = \text{const} \quad [8]$$

$x = s(t)$ is the surface of the bar which is receding with a velocity $\dot{s}(t)$ as measured in a laboratory coordinate system. This velocity is obtained from a heat balance at the surface:

$$\dot{s}(t) = \frac{\dot{q}_{0r} + [k(\partial T/\partial x)]_{x=s}}{\rho\psi} \quad [9]$$

where ψ is the heat absorbed per unit mass ablated for small ablation rates:

$$\psi = \Delta h_v + \eta(H_\infty - H_w) \quad [10]$$

Transforming from the laboratory coordinate system to the ablating surface, let

$$y = (x - s) \quad [11]$$

Eq. [3] transforms as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \rho c\dot{s} \frac{\partial T}{\partial y} + aF_0 e^{-ay} \quad [12]$$

where

$$F_0 = (1 - R)F_x(t) \quad [13]$$

The initial and boundary conditions become

$$T(y, 0) = T_0 \quad [14]$$

$$T(0, t) = T_A \quad [15]$$

$$T(\infty, t) = T_0 \quad [16]$$

The ablation velocity becomes

$$\dot{s} = \frac{\dot{q}_{0r} + [k(\partial T/\partial y)]_{y=0}}{\rho\psi} \quad [17]$$

Steady-State Solution

In the steady-state limit, with constant thermal and optical properties, Eq. [12] reduces to the following:

$$\frac{d^2T}{dy^2} + \frac{\dot{s}}{\alpha}\frac{dT}{dy} = -a\frac{F_0}{k}e^{-ay} \quad [18]$$

where α represents the thermal diffusivity and F_0 is constant. The first integral of [18] is obtained immediately:

$$\frac{dT}{dy} + \frac{\dot{s}}{\alpha}T = \frac{F_0}{k}e^{-ay} + K_1 \quad [19]$$

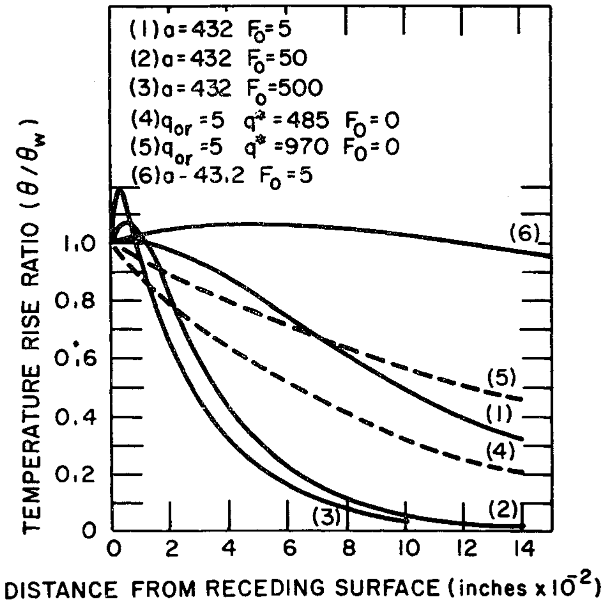

Fig. 3 Steady-state ablation: radiant heating

By inspection, an integrating factor of [19] is seen to be $\dot{s}y/\alpha$. Consequently, the solution is

$$T = \frac{F_0}{k(\dot{s}/\alpha - a)}\, e^{-ay} + K_2 e^{-(\dot{s}/\alpha)y} + K_1 \qquad [20]$$

Applying the boundary conditions and defining the temperature rise by

$$\theta = (T - T_0) \qquad [21]$$

the solution to [18] is found to be the following:

$$\frac{\theta}{\theta_w} = e^{-(\dot{s}/\alpha)y} + \frac{F_0}{k\theta_w(a - \dot{s}/\alpha)}\left[e^{-(\dot{s}/\alpha)y} - e^{-ay}\right] \qquad 0 < a \qquad [22]$$

To use [22], one must evaluate $\dot{s}$ from [17]. This requires evaluating $[k(dT/dy)]_{y=0}$. Differentiating [22]

$$\frac{d\theta}{dy} = -\frac{\dot{s}}{\alpha}\,\theta_w e^{-(\dot{s}/\alpha)y} + \frac{F_0}{k(a - \dot{s}/\alpha)}\left[ae^{-ay} - \frac{\dot{s}}{\alpha}\,e^{-(\dot{s}/\alpha)y}\right] \qquad [23]$$

Thus at $y = 0$

$$\left(\frac{dT}{dy}\right)_{y=0} = \frac{F_0}{k} - \frac{\dot{s}}{\alpha}\,\theta_w \qquad [24]$$

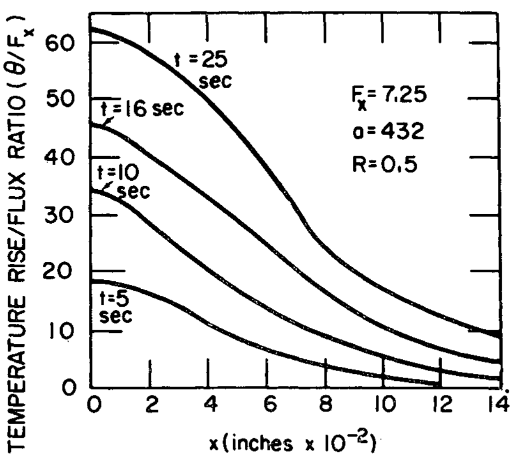

Fig. 4 Transient response: radiant heating

Combining [17] and [24], $\dot{s}$ is found to be the following:

$$\dot{s} = \frac{\dot{q}_{0r} + F_0}{\rho(c\theta_w + \psi)} \qquad [25]$$

Eqs. [22] and [25] completely specify the steady-state temperature distribution in a semi-infinite bar heated by constant radiative and aerodynamic inputs for constant thermal and optical properties. The first term on the right side of [22] is the usual steady-state temperature distribution due to the aerodynamic heating, whereas the last term is the perturbation due to the radiant input. The ablation rate given in [25] is in the usual form:

$$\dot{s} = \dot{q}/\rho q^* \qquad [26]$$

with the understanding that $\dot{q}$ is the sum of the aerodynamic and radiant heat inputs. Eq. [24] shows that the surface gradient will be positive provided

$$(\dot{s}/\alpha)\theta_w < F_0/k \qquad [27]$$

In particular, if one has pure radiant heating

$$\left(\frac{dT}{dy}\right)_{y=0} = \frac{F_0}{k}\left(1 - \frac{c}{c\theta_w + \psi}\right) \qquad [28]$$

The second term in the parentheses is less than unity; thus a positive surface gradient always exists in steady-state ablation for pure radiant heating. The value of y at which this maximum occurs is

$$y_{\max} = \frac{1}{(\dot{s}/\alpha - a)} \ln\left\{\frac{\dot{s}/\alpha}{a}\left[1 + \left(a - \frac{\dot{s}}{\alpha}\right)\frac{k\theta_w}{F_0}\right]\right\} \qquad [29]$$

Transient Solution

In the nonablating transient case with constant properties and with $s = 0$, Eq. [3] reduces to

$$\frac{\partial T}{\partial t} = \alpha\,\frac{\partial^2 T}{\partial x^2} + \frac{a(1 - R)}{\rho c}\,F_x e^{-ax} \qquad [30]$$

A solution to this equation was obtained for the case of F_x independent of time and for the following initial and boundary conditions. See Ref. 12 for solution with a constant surface temperature:

$$T(x, 0) = T_0 = \text{const} \qquad [31]$$

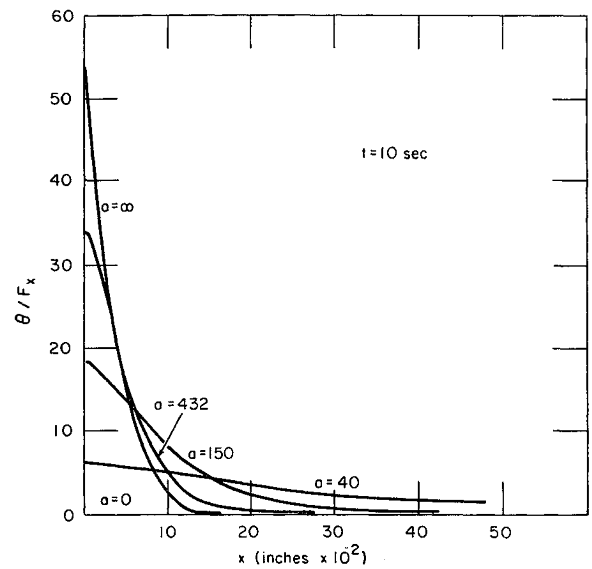

Fig. 5 Variations of temperature profiles with attenuation coefficient

$$\left(\frac{\partial T}{\partial x}\right)_{x=0} = -\frac{\dot{q}_c}{k} = \text{const} \quad [32]$$

$$\lim_{x\to\infty} \frac{\partial T}{\partial x} = 0 \quad [33]$$

Applying the Laplace transformation, $T(x,t) \to \tau(x,\phi)$ and [30] becomes an ordinary differential equation:

$$\frac{d^2\tau}{dx^2} - \frac{\phi}{\alpha}\tau = -\frac{T_0}{\alpha} - \frac{a(1-R)}{k\phi}F_x e^{-ax} \quad [34]$$

Solution of [34] subject to boundary conditions [32] and [33], appropriately transformed, gives

$$\tau(x,\phi) = \left[\frac{-a^2(1-R)F_x}{\rho c\phi(\phi - a^2\alpha)} + \frac{1}{\phi}\frac{\dot{q}_c}{k}\right](\alpha/\phi)^{1/2} \times \exp[-x(\phi/\alpha)^{1/2}] + \frac{a(1-R)F_x\exp(-ax)}{\rho c\phi(\phi - a^2\alpha)} + \frac{T_0}{\phi} \quad [35]$$

Taking the inverse transformation of [35] and recalling that $\theta = (T - T_0)$, the solution is found to be the following:

$$\theta(x,t) = \frac{(1-R)F_x}{2ak}\left\{\exp(ax + a^2\alpha t) \times \operatorname{erfc}\left[a(\alpha t)^{1/2} + \frac{x}{2(\alpha t)^{1/2}}\right] - \exp(-ax + a^2\alpha t) \times \operatorname{erfc}\left[-a(\alpha t)^{1/2} + \frac{x}{2(\alpha t)^{1/2}}\right]\right\} + \left[\frac{(1-R)F_x}{(k\rho c)^{1/2}} + \frac{\dot{q}_c}{(k\rho c)^{1/2}}\right] \times \left\{2(t/\pi)^{1/2}\exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{x}{\alpha^{1/2}}\operatorname{erfc}\left[\frac{x}{2(\alpha t)^{1/2}}\right]\right\} + \frac{(1-R)F_x}{ak}\exp(-ax)\{\exp(a^2\alpha t) - 1\} \quad [36]$$

Eq. [36] can be used to calculate the temperature history at any point of a semi-infinite bar subject to boundary conditions [31–33]. This solution is valid up to the time when the surface reaches the ablation temperature T_A. The solution can readily be extended by means of Duhamel's theorem to the case of time dependent F and $\dot{q}_c$.

Numerical Results

Steady-State Ablation

For a sample calculation, consider the case of a semi-infinite bar in steady-state ablation due to a pure radiative heat input. The constant properties used in the calculation are as follows:

k = 3.46 × 10^{-5} Btu/ft-sec-°F
ρ = 70 lb/ft³
c = 0.46 Btu/lb-°F
T_A = 900°F
T_0 = 70°F
q^* = 485 Btu/lb
a = 432 ft^{-1}

Combining [22] and [26], the temperature distribution based upon these properties is given as follows for y in feet and F_0 in Btu/ft²-sec:

$$\frac{\theta}{\theta_w} = \exp(-27.4\, F_0 y) + \frac{35\, F_0}{(432 - 27.4\, F_0)} \times [\exp(-27.4\, y) - \exp(-432\, y)] \quad [37]$$

The results of a parametric study of this solution are given in Fig. 3. The curves labelled 1, 2, and 3 show the temperature distribution calculated for F_0 = 5, 50, and 500, respectively. The positive surface gradient found for pure radiant heating is clearly illustrated. For comparison with curve 1, the corresponding distribution for pure aerodynamic heating of the same net amount, i.e., $\dot{q}_{0_r} = F_0 = 5$, and for the same q^* is shown by the dotted curve labelled 4. For these conditions, the radiant mode of heating causes a greater temperature response than the convective mode. In actuality, however, q^* is never the same for aerodynamic heating as it is for radiant heating due to the transpiration contribution that is absent for pure radiant heating. Accordingly, the pure aerodynamic heating case represented by curve 4 of Fig. 3 was recomputed, using twice the value of q^*. The result is given by curve 5. It is thought that curves 1 and 5 serve as a better comparison of the relative effectiveness of radiant and aerodynamic heating. Generally speaking, for the same net input, i.e., for $\dot{q}_{0r} = F_0$, a greater ablation rate results from radiant heating than for aerodynamic. This is due to the lower q^* value associated with the pure radiant input. Also, the radiant input causes a greater temperature response near the surface than does aerodynamic heating.

The effect of decreasing a is shown by curve 6 of Fig. 3. In computing these values, a was reduced by an order of magnitude from the value used for curve 1. As shown, the temperature penetration is increased enormously by decreasing a. Increasing the value of a, on the other hand, causes less penetration. In the limit of a strongly absorbing material, such as a metal, a tends toward infinity, and curve 1 approaches the limiting value shown by curve 4.

Transient Pre-Ablation Response

A plot of Eq. [36] is given in Fig. 4 for pure radiant heating of a bar with the following properties:

k = 3.61 × 10^{-5} Btu/ft-sec-°F
ρ = 70.6 lbm/ft³
c = 0.42 Btu/lbm-°F
a = 432 ft^{-1}
F_x = 7.25 Btu/ft²-sec
R = 0.5

The ordinate of this figure represents the temperature rise in degrees Fahrenheit divided by the constant incident flux F_x. The abscissa is the position coordinate x in inches. The parameter is time t in seconds.

Fig. 5 illustrates the effect of variation of flux attenuation coefficient upon the temperature response of the system under influence of a pure radiative flux. Mathematically and physically it can be shown that

$$\lim_{a\to\infty} \theta = \frac{F_0}{(k\rho c)^{1/2}}\left\{2(t/\pi)^{1/2}\exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{x}{\alpha^{1/2}}\operatorname{erfc}\left[\frac{x}{2(\alpha t)^{1/2}}\right]\right\} \quad [38]$$

As can be seen from Fig. 5, the net effect of increasing the flux attenuation coefficient is to increase surface temperature and to decrease penetration. In the steady-state solution, the same effect is found in regard to penetration.

Discussion

Among the specific results obtained, some deserve particular attention. It is shown that during ablation, in a combined radiant and convective heating environment, it is possible to obtain a positive temperature gradient at the surface. If the heat input is purely radiant, a positive gradient is actually required to supply the energy needed for ablation, which is assumed to occur in the mathematical plane of the surface. Since radiant energy can only be deposited in a finite volume (and not in a mathematical plane), this energy must be supplied from somewhere in order that the ablation may be maintained. This energy can arrive at the surface from the interior of the material only by conduction. This requires the temperature on the interior of the material to be greater than that of the surface; thus a positive gradient is required in order to maintain the ablation. It is also shown that, for pure radiative heating, the steady-state ablation rate is greater than that for a net convective input $\dot{q}_{0r}$ numerically equal to the magnitude F_0 of the radiant input. This is due to the lower value of q^* appropriate to radiant heating.

A direct comparison of the effectiveness of radiant vs convective heating upon the temperature response of a material must take into consideration the thermal and optical parameters of the material together with the enthalpy appropriate to the convective environment. For steady-state ablation, a typical comparison is shown by curves 1 and 5 of Fig. 3. The convective heating associated with curve 5 is based upon a nominal value of q^* twice that used for the radiative heating shown by curve 1. This difference in q^* arises from the $(\eta\Delta H)$ term in the q^* for convective heating which is not present for the radiant case. As shown by these two curves, radiant heating causes a greater temperature response of layers close to the ablating surface. An inversion of this effect occurs at $x \doteq 0.072$ in. for the particular case illustrated. Decreasing the value of a moves this point further into the material, as indicated by curve 6. The effect of increasing the value of a is to cause curve 1 to approach curve 4 as a limiting case of pure radiative heating of a metallic-like absorber. For such a case, radiant heating results in less temperature penetration than the corresponding convective case.

The effect of varying the value of the radiant flux attenuation coefficient for pure radiant heating in the nonablating transient case is shown in Fig. 5. The effect of increasing a is to raise the surface temperature at a given instant of time, and vice versa. It can be concluded that a material with a low value of a will require a longer time to reach the ablation temperature than one with a higher value. The temperature penetration is shown to be greater, at any instant of time, for materials with the lower value of a than for materials with a higher attenuation coefficient.

Summary and Conclusions

A method is presented for calculating the temperature response of a semitransparent material heated by a combined radiant and convective input. The formulation assumes that the attenuation of radiant flux in the material follows an exponential decay law and neglects the effect of reflection from rear boundary.

Application is made to a semi-infinite bar heated by a mixture of radiant and convective inputs that do not vary with time. Two analytic solutions are given, one for steady-state ablation and the other for the nonablating transient response. All properties are regarded as constants. These solutions are useful in providing a basis for understanding the heat transfer phenomena occurring in a combined convective and radiative heating environment, and they provide a functional means of checking computer solutions of such problems.

From a thermal protection system point of view, it is clear that it is desirable to develop materials with as high an a as possible in order to provide suitable thermal protection from a radiant heating environment. One technique that may be used to accomplish this is to opacify the materials by the addition of large numbers of scattering centers. The effect of this is to force the photons to travel a very long optical path in traversing the material. Thus, even if the optical absorption coefficient is small, the attenuation of the radiant flux is large due to the long path traveled.

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